of the unsteady analogy of hypersonic small-deflection theory whereby the cylindrical unsteady flow becomes analogous to the flow of a high Mach number jet to vacuum.

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Experimentally Determined Reynolds Analogy Factors for Flat Plates

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In fluid mechanics, the analogy between skin friction and heat-transfer coefficients is remarkable in its simplicity and wide range of usefulness. The original analogy formulated by Reynolds in 1874 is now usually written

$$C_h = C_f/2 \tag{1}$$

and is based on the assumption that momentum and heat are transferred by similar processes. Because such a relationship depends upon the relative diffusion of momentum and heat through the boundary layer, the effect of the Prandtl number must be taken into account in order to provide general results. For a smooth, flat surface, it is common to write

$$C_h = (1/S)(C_f/2) \tag{2}$$

where S is designated the Reynolds analogy factor and is a strong function of the Prandtl number. For the low-speed, turbulent-flow case, Colburn has shown that the modified Reynolds analogy

$$C_h = (1/P_r^{2/3})(C_f/2) \tag{3}$$

properly predicts the variation of S with P_r; this relation has been found to give good results for air with Reynolds numbers of less than 10^6.[1] The expression, of course, reduces to the simple Reynolds analogy for $P_r = 1$. Several modifications[1–4] to the Reynolds analogy have been devised to account for the effects of compressibility and dissipation in high-speed flow.

Extensive theoretical and experimental studies have been reported on the Reynolds analogy for various surfaces and flow conditions, but there appears to be little experimental information available for the case of compressible turbulent flow over rough surfaces. The purpose of this note is to present some recent results from the simultaneous measurement of local skin-friction and heat-transfer rate at adjacent positions on a flat-plate model. The measurements were made with both smooth and rough plate surfaces, as part of a general study of the influence of roughness of skin friction and heat transfer in turbulent, compressible flows.

Experimental Apparatus

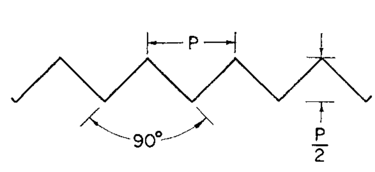
An intermittent flow wind tunnel having a 6 × 7-in. test section and fixed nozzle blocks was employed for the experiments. The nominal Mach number was 4.93, the stilling-chamber pressure was 255 psia, and a stagnation temperature range of 620–1100°R was employed. The corresponding range of Reynolds number was 0.5 to 1.5 × 10^7/ft. The nominal plate temperature was 555°R, resulting in a range of wall-to-freestream temperature ratio of 2.9 to 5.2. A more complete description of the tunnel and its associated equipment is given in Ref. 5.

The basic flat-plate model was 6 in. wide, 1 in. thick, and 17.5 in. long. A 15° wedge angle was used for the plate nose. The plate body was constructed of copper with integral cooling passages to maintain a constant temperature during each run. The model spanned a test section with the flat test surface facing downward. A slug-type calorimeter and a floating-element skin-friction balance, both employing 1-in.-diam disks, were located 12.5 in. aft of the leading edge. The 20-g, 0.150-in.-thick copper calorimeter was insulated from the plate by an air space, except where it was secured in its recess by a thin annular ring of epoxy cement. An iron-constantan thermocouple embedded in the calorimeter was used to continuously monitor calorimeter temperature, which was indicated by a recording potentiometer. The copper skin-friction balance floating element was mounted flush with the test surface in circular opening, which provided a 0.002-in. radial clearance between the element and the surrounding plate. A linear variable-differential transformer was used to measure the deflection of the element by friction forces. A second recording potentiometer was used to continuously record the balance output. The balance operating principles are detailed in Ref. 6. Three pressure orifices were connected to mercury manometers to measure static pressure on the test surface. Five iron-constantan thermocouples were embedded just below the test surface along the plate longitudinal centerline to determine plate-temperature distributions.

The first test model had a smooth surface, whereas each of the succeeding three had uniformly rough surfaces consisting of 90° V grooves oriented transversely to the flow direction. The roughness pitch dimensions were 0.005, 0.010, and 0.030 in. (Fig. 1). The 90° groove angle resulted in a groove depth that was approximately one-half of the pitch. Copper, which was selected for the plate body to minimize temperature gradients, proved too soft to machine the V grooves directly into the test surface. The desired roughness was obtained on the plate surface, floating-element, and calorimeter disks by first coating them with thin layers of tin-lead solder into which the grooves were then pressed, using a steel roller having several grooves machined into its periphery. The 2-in.-diam roller was installed in a machine-shop shaper in place of the usual cutting bit to roll the grooves in the test surface. This technique produced a highly uniform test surface free of ragged edges and irregularities. A strip extending back 1 in. from the plate leading edge was left smooth on each model; a one-half-in.-wide tripper strip of No. 80 grit cloth was secured across the rear half of this region.

For all tests, the plate temperature was maintained at approximately 555°R by cooling-water regulation. Variations in heat-transfer rates were obtained by selecting several tunnel stagnation temperatures. For runs during which heat transfer was measured, as flow conditions stabilized, water from a retractable probe was directed to the calorimeter disk in order to depress its temperature below that of the surrounding plate. After probe retraction, the calorimeter temperature increased rapidly toward the flow recovery temperature. The slope of the temperature record, at the instant when the calorimeter and plate temperatures were equal,

Fig. 1 Cross - section of surface roughness.



Received February 15, 1965. This work was sponsored by the U. S. Navy Bureau of Weapons under Contract No. NOrd-16498.

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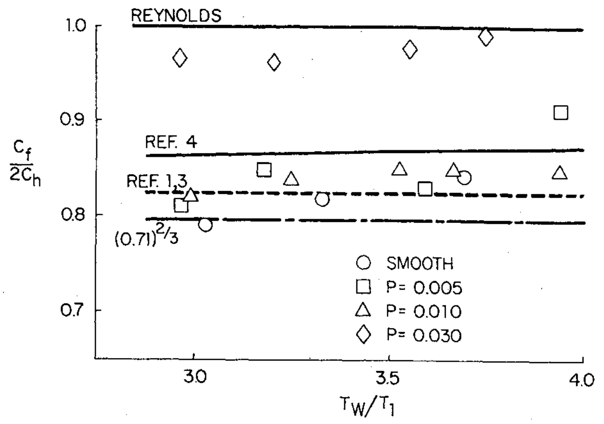

Fig. 2 Comparison of theory and test results.

was used to calculate the heat-transfer coefficient in the conventional manner, which is described in detail in Ref. 7. A recovery factor of 0.88 was used to determine the adiabatic-wall temperature.

The skin-friction balance output remained substantially constant during the period of stabilized tunnel flow, as determined from the potentiometer recordings made at 2-sec intervals. After each run, the entire balance system was calibrated using dead weights to determine the friction force corresponding to the recorded balance output. The skin-friction coefficient was then calculated using the floating-element area and freestream dynamic pressure. The experimental values of skin-friction and heat-transfer coefficients were then used to compute the Reynolds analogy factor using Eq. (2).

Results

The accuracy of the measured skin-friction and heat-transfer coefficients obtained from the smooth-plate tests was evaluated by comparison with the theory of Van Driest[3]; good agreement was obtained in both cases. For the tests involving surface roughness, it is convenient to use a nondimensional roughness height η_k defined by

$$\eta_k = (U_\tau/\nu_w)(P/2)$$

where $P/2$ is the depth of the V grooves. The thickness of the laminar sublayer on a flat plate in turbulent flow may also be expressed in terms of η; the values range from 5.0 to 11.5, depending on the rate of heat transfer. Several investigators have suggested that a surface be considered aerodynamically smooth if its roughness height is less than the sublayer thickness. For the 0.005-in. pitch of the present tests, η_k ranged from 6.0 to 6.6; since these values are well below the corresponding sublayer thickness, the surface theoretically would be considered smooth. This was confirmed by the test results, which were essentially the same as with the smooth surface.

The values of η_k based on groove depth for the 0.010-in. pitch were 12.9 to 13.4; these are larger than the smooth-plate sublayer thickness, thus indicating the surface was not theoretically smooth. The test values of C_h and C_f were significantly greater than for the smooth plate, demonstrating that it was also not smooth in the experimental sense. Similarly, the 0.030-in. pitch V grooves (η_k of 43.6–47.4) were both experimentally and theoretically rough.

The Reynolds number range of the test, based on the distance from the leading edge to the test station, was 5.0 to 7.5×10^6. Because of this limited range and the constant Mach number, the heating rate was determined primarily by the wall-to-freestream temperature ratio T_w/T_1. As previously noted, T_w was constant, and T_1 was determined by selection of the tunnel stagnation temperature. Thus, the over-all result was that a unique Reynolds number was obtained for each T_w/T_1. At each value of T_w/T_1, from three to seven runs were made during which simultaneous heating and friction measurements were obtained; these results were averaged to give a single data point for that value of T_w/T_1.

The experimental Reynolds analogy factors are presented in Fig. 2 as a function of T_w/T_1; several theoretical values, all for a smooth surface, are also shown for comparison. In each case, the theoretical values were calculated using the test conditions corresponding to each experimental data point; hence, a direct comparison may be made. The general trend of the data indicates that S decreases with increasing heat transfer, which is in accord with Ref. 4. The scatter in the data makes very precise comparisons unwarranted, but it appears that no consistent roughness effect is apparent for the 0.005- and 0.010-in. roughness. The theory of Rubesin[1] is almost identical to that of Van Driest[3] for the present test conditions; hence, no attempt has been made to distinguish between them (Fig. 2). The theories of Refs. 1, 3, and 4 all agree with the experimental results to within 5%, the prediction of Ref. 4 being higher than the bulk of the test data and the other two predictions being somewhat on the low side. It should be noted that the theory shown in Ref. 3 is the value of $S = 0.825$, which is recommended therein for engineering calculations. This value is a compromise selected to cover a comparatively broad range of Mach numbers and heating rates; hence the agreement with experiment is even more remarkable. The Colburn relation of $P_r^{2/3}$ is somewhat lower than the test data, but the agreement is considered good in view of the extension from Colburn's original low-speed and low-Reynolds-number conditions.

Although the Reynolds analogy factor obtained with a roughness pitch of 0.010 in. did not differ significantly from that obtained with a smooth surface, the skin-friction and heat-transfer rates did differ. For this roughness, the skin-friction and heat-transfer coefficients were both larger than for the smooth surface; however, the increases were in the same proportion, with the result that S was unchanged. This was not true for the roughest surface ($P = 0.030$ in.), however. Here increases in C_f and C_h were also noted, but the increase in C_f was proportionately higher. Thus, as seen in Fig. 2, the Reynolds analogy factor for the roughest surface was appreciably higher than for the others. The value obtained agrees quite well with the original analogy of Reynolds; this agreement is probably fortuitous, however, as increasing roughness beyond this level might be expected to produce even higher values of S.

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